Assume that B is an adapted process with finite first variation. For simplicity, assume further that B has continuous trajectories. What we did in class is

$$[B,B]_T = \lim_{\|P\|\to 0} \sum_{k=0}^{n-1} (B_{t_{k+1}} - B_{t_k})^2 = \lim_{\|P\|\to 0} \sum_{k=0}^{n-1} |B_{t_{k+1}} - B_{t_k}| \cdot |B_{t_{k+1}} - B_{t_k}|$$
$$\leq \lim_{\|P\|\to 0} \left(\sum_{k=0}^{n-1} |B_{t_{k+1}} - B_{t_k}| \right) \left(\max_{0\leq k\leq n-1} |B_{t_{k+1}} - B_{t_k}| \right).$$

Then, I wrote

$$\|P\| = \max_{0\leq k\leq n-1} |B_{t_{k+1}} - B_{t_k}|,$$

which is **WRONG**! The correct thing is

$$\|P\| = \max_{0\leq k\leq n-1} |t_{k+1} - t_k|.$$

Don't worry, the rest of the proof can be easily fixed. We still have

$$\lim_{\|P\|\to 0} \sum_{k=0}^{n-1} |B_{t_{k+1}} - B_{t_k}| = V_{[0,T]}(B) < \infty.$$

For the second term, since B_t is uniformly continuous on $[0,T]$, we have

$$\lim_{\|P\|\to 0} \max_{0\leq k\leq n-1} |B_{t_{k+1}} - B_{t_k}| = 0.$$

Combining these two together gives us $[B,B]_T = 0$.